Διαγώνισμα *Γ Τάξης Ενιαίου Λυκείου*

Ταλαντώσεις - Κύματα

Σύνολο Σελίδων: εννέα (9) - Διάρκεια Εξέτασης: 3 ώρες

Κυριακή 10 Δεκεμβρίου 2023

Ονοματεπώνυμο: **#frontistiri**

Θέμα Α

Α.1 → (α)

Α.2 → (β)

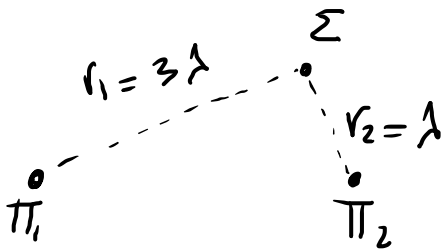
Α.3 → (γ)

Α.4 → (γ)

Α.5 → Λ, Λ, Λ, Σ, Λ

Β.1 → **(β)**

$r_1 - r_2 = 2\lambda$ είναι ενίσχυση



απόσβεση → $r_1 - r_2 = (2N+1)\frac{\lambda'}{2}$ $N = 0, 1, \ldots$

$\Rightarrow 2\lambda = (2N+1)\frac{\lambda'}{2}$ $\quad u_\delta = \lambda \cdot f$

$\Rightarrow 2 \cdot \frac{u_\delta}{f} = (2N+1)\cdot\frac{u_\delta}{2f'} \Rightarrow f' = \frac{(2N+1)}{4} \cdot f$

ελάχιση συχνότητα $N = 0$ $\Big\}$ $f' = \frac{f}{4}$ οπότε $\frac{f'-f}{f}\cdot 100\% = -75\%$

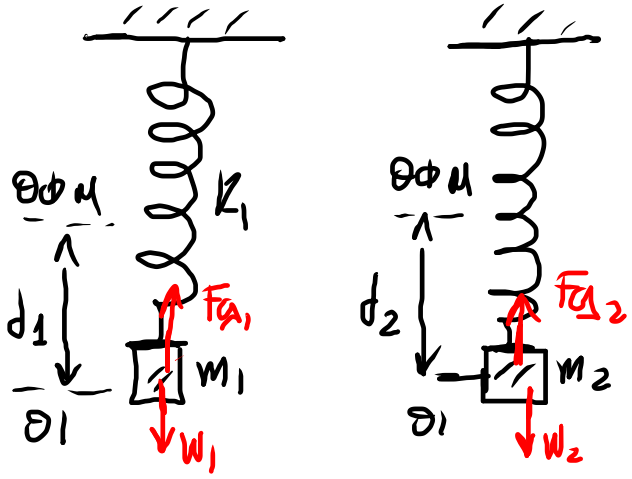
Β.2 → **(α)**

$L = \frac{\lambda_1}{4} + 2\frac{\lambda_1}{2} \Rightarrow L = \frac{5\lambda_1}{4}$

$L = \frac{\lambda_2}{4} + 4\frac{\lambda_2}{2} \Rightarrow L = \frac{9\lambda_2}{4}$

άρα $\frac{5\lambda_1}{4} = \frac{9\lambda_2}{4} \Rightarrow 5\frac{u_\delta}{f_1} = 9\frac{u_\delta}{f_2} \Rightarrow f_2 = \frac{9}{5} f_1$

Β.3 → (β)



$K_1 = 4K_2$, $d_2 = 2d_1$

Ισορροπία → $\Sigma F = 0$

$F_{ελ} = w$

$$\left.\begin{aligned} K_1 d_1 &= m_1 g \\ K_2 d_2 &= m_2 g \end{aligned}\right\} \frac{K_1 d_1}{K_2 d_2} = \frac{m_1 g}{m_2 g} \Rightarrow$$

$$\Rightarrow 2 = \frac{m_1}{m_2} \Rightarrow \underline{m_1 = 2m_2}$$

[Αυραία → Θ1Τ

$\Delta t = \frac{T}{4}$, $T = \frac{2\pi}{\omega}$

$D = m\omega^2 \Rightarrow \omega = \sqrt{\frac{D}{m}}$]

$$\frac{\Delta t_1}{\Delta t_2} = \frac{\frac{T_1}{4}}{\frac{T_2}{4}} = \frac{T_1}{T_2} = \frac{\frac{2\pi}{\omega_1}}{\frac{2\pi}{\omega_2}} = \frac{\omega_2}{\omega_1} = \frac{\sqrt{\frac{K_2}{m_2}}}{\sqrt{\frac{K_1}{m_1}}} = \sqrt{\frac{K_2 \cdot m_1}{K_1 m_2}} \Rightarrow \underline{\frac{\Delta t_1}{\Delta t_2} = \frac{1}{\sqrt{2}}}$$

Θέμα Γ $y = 0{,}4\eta\mu(\pi t)\ (SI) \rightarrow A = 0{,}4m,\ \omega = \pi\ r/s$

$v_\delta = \lambda \cdot f \Rightarrow 0{,}2 = \lambda \cdot 0{,}5 \Rightarrow \underline{\lambda = 0{,}4m}$

$\omega = 2\pi f \Rightarrow \underline{f = 0{,}5\,Hz}$ ↓ $T = 2s$

Γ.1 $y = A\eta\mu 2\pi\left(f \cdot t - \frac{x}{\lambda}\right) = 0{,}4\,\eta\mu 2\pi\left(0{,}5t - \frac{x}{0{,}4}\right)$

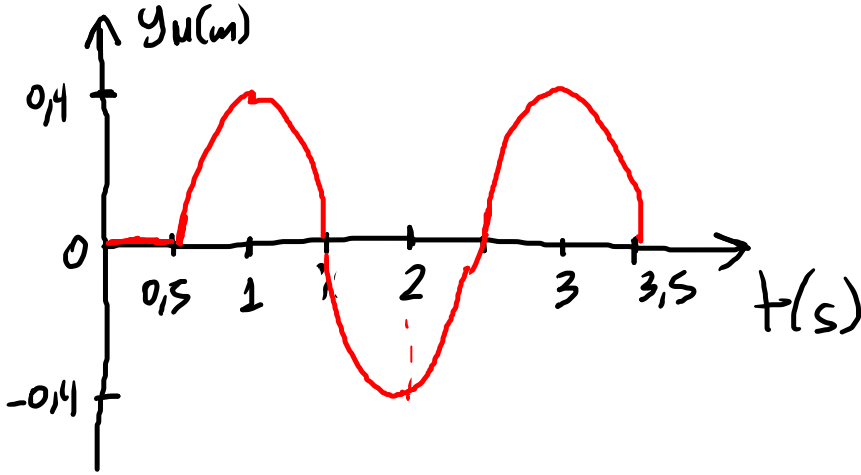
$\Rightarrow \underline{y = 0{,}4\eta\mu(\pi t - 5\pi x)\ (SI)}$

Γ.2 δημιο $M\ (x_M = 0{,}1m)$

$\underline{y_M = 0{,}4\eta\mu(\pi t - 0{,}5\pi)\ (SI)}$

$\varphi \geq 0 \Rightarrow \pi t - 0{,}5\pi \geq 0$, $\underline{t \geq 0{,}5s}$

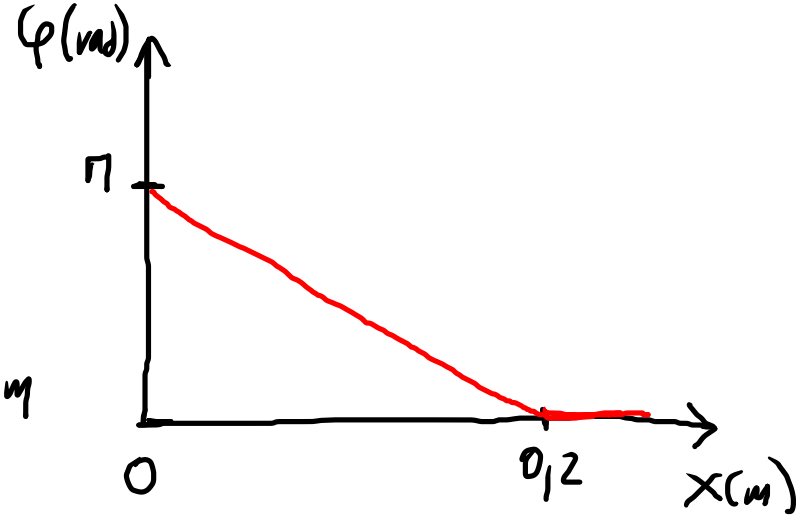
* η $t \geq t_M$, $t_M = \frac{x_M}{v_\delta}$



Γ.3 $\varphi = \pi t - 5\pi x$

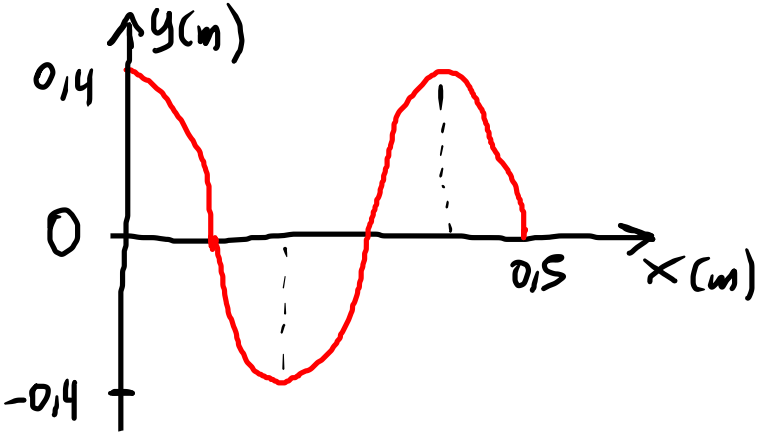
για $t = 1s$ $\quad \varphi(1) = \pi - 5\pi x \ (SI)$

$\varphi \geq 0 \Rightarrow \pi - 5\pi x \geq 0 \Rightarrow x \leq 0{,}2m$

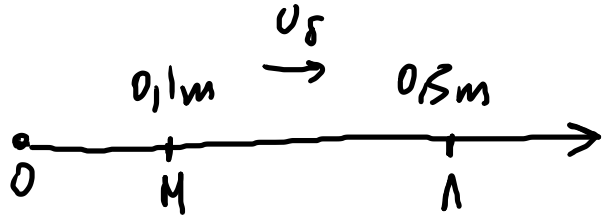


Γ.4 για $t = 2{,}5\,s \rightarrow x = v_\delta t = 0{,}5m$

άρα $x = \lambda + \frac{\lambda}{4}$



Γ.5



$y_M = 0$
$v_M > 0$

$$\Delta\varphi = \varphi_M - \varphi_\Lambda = 2\pi\left(ft - \frac{x_M}{\lambda}\right) - 2\pi\left(ft - \frac{x_\Lambda}{\lambda}\right)$$

$$\Delta\varphi = \frac{2\pi}{\lambda}(x_\Lambda - x_M) \Rightarrow \underline{\Delta\varphi = 2\pi \text{ rad}}$$

$y_M = A\eta\mu\varphi_M = 0 \Rightarrow \eta\mu\varphi_M = 0 \Rightarrow \varphi_M = 2k\pi$ ή $\varphi_M = 2k\pi + \pi$

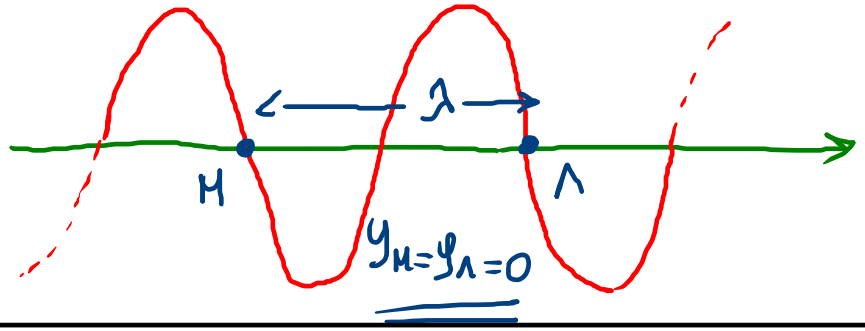
$V_M > 0 \Rightarrow \sigma\upsilon\nu\varphi_M > 0 \rightarrow \varphi_M = 2k\pi$

Οπότε $\varphi_M - \varphi_\Lambda = 2\pi \Rightarrow \varphi_\Lambda = 2k\pi - 2\pi$

Άρα $y_\Lambda = A\eta\mu\varphi_\Lambda = A\eta\mu(2k\pi - 2\pi) \Rightarrow \underline{y_\Lambda = 0}$

(*) Ενα Μακρια αφου $\Delta\varphi = 2\pi \rightarrow$ ειναι σε συμφωνια φασης

Αρα $y_M = y_\Lambda$ καθε στιγμη $\Rightarrow y_\Lambda = 0$

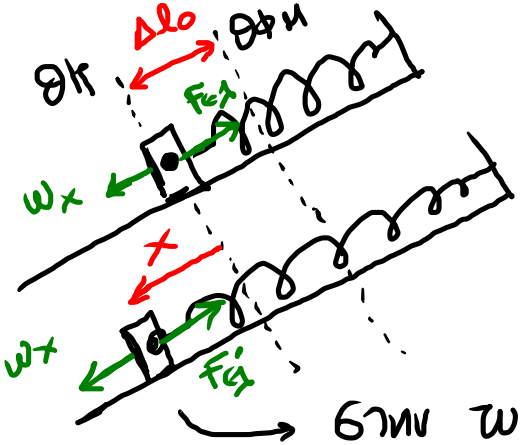


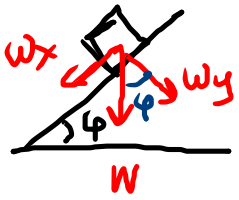
Θέμα Δ

$m = 1\,kg$, $M = 3\,kg$, $K = 100\,N/m$, $\varphi = 30^\circ$, $\Delta P = \frac{2\sqrt{6}}{3}\,kg\cdot m/s$

Δ.1

Ταλάντωση του συσσωματώματος





ΘΙΤ: $\Sigma F_x = 0$

$F_{ελ} = w_x \Rightarrow K\Delta l_0 = w_x$ (1)

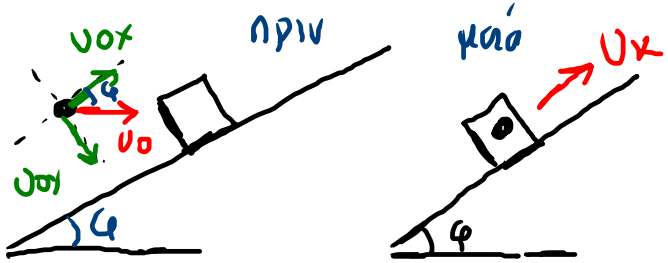
στη τυχαία θέση: $\Sigma F_x = w_x - F_{ελ}'$

$\Sigma F_x = w_x - K\cdot(\Delta l_0 + x) = w_x - K\Delta l_0 - Kx$

$\overset{(1)}{\Rightarrow}$ $\Sigma F_x = -Kx$ άρα με $D = K$

$D = K = (m+M)\omega^2 \Rightarrow \omega = \sqrt{\frac{K}{M+m}} \Rightarrow \omega = 5\,r/s$ οπότε $T = \frac{2\pi}{\omega} = \frac{2\pi}{5}\,s$

Δ.2



ΑΔΟ στον $x'Ox$

$$\vec{P}_{\sigma\upsilon\sigma(x)}^{\pi\rho\iota\nu} = \vec{P}_{\sigma\upsilon\sigma(x)}^{\mu\epsilon\tau\acute{\alpha}}$$

$$m\upsilon_{ox} = (m+M)\upsilon_K$$

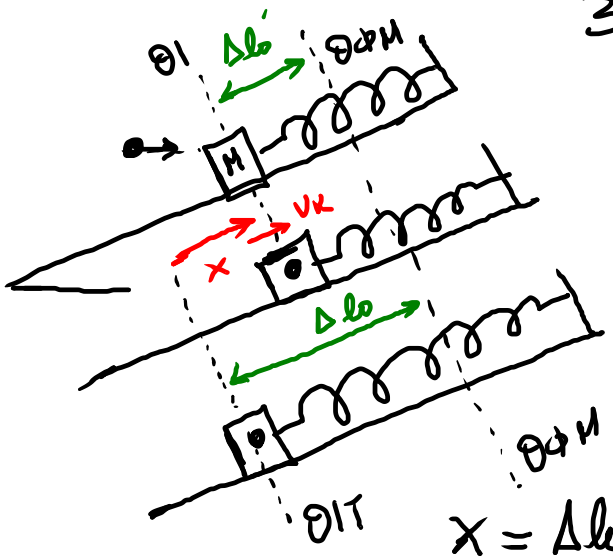
$$\upsilon_{ox} = 4\upsilon_K \Rightarrow \upsilon_o \sigma\upsilon\nu 30 = 4\upsilon_K$$

$$\Delta\vec{P} = (\Delta P_x, \Delta P_y)$$

$$\Delta P_x = 0 \quad , \quad \Delta P = \Delta P_y = 0 - m\upsilon_{oy}$$

$$|\Delta P| = m \cdot \upsilon_o \cdot \eta\mu\varphi \Rightarrow \frac{2\sqrt{6}}{3} = \upsilon_o \cdot \frac{1}{2} \Rightarrow \upsilon_o = \frac{4}{3}\sqrt{6}\, m/s$$

$$\begin{cases} \frac{4}{3}\sqrt{6}\cdot\frac{\sqrt{3}}{2} = 4\upsilon_K \\ \upsilon_K = \frac{\sqrt{2}}{2}\, m/s \end{cases}$$



$$x = \Delta l_o - \Delta l_o'$$

ΑΔΕΤ στην θέση που έχει η μπάλα

$$E = K + U \Rightarrow \frac{1}{2}DA^2 = \frac{1}{2}(m+M)\upsilon_K^2 + \frac{1}{2}Dx^2$$

$$\Rightarrow A^2 = \frac{m+M}{K}\upsilon_K^2 + x^2 \quad (2)$$

Από (1) $\Rightarrow K\Delta l_0 = (M+m)g\eta\mu 30 \Rightarrow \Delta l_0 = 0,2\,m$

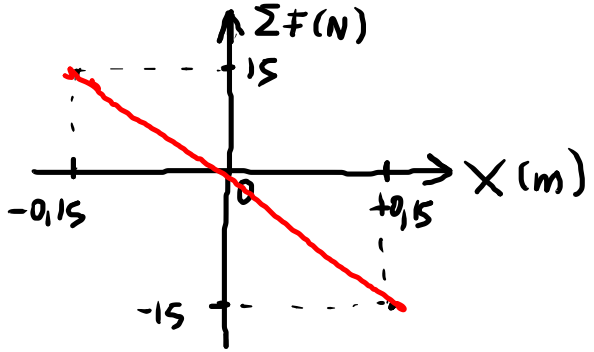
Στη θ.ι του Μ : $\Sigma F_x = 0 \Rightarrow K\Delta l_0' = Mg\eta\mu 30 \Rightarrow \Delta l_0' = 0,15\,m$

Οπότε $|x| = 0,2 - 0,15 = 0,05\,m$

(2) $\rightarrow$ $A^2 = \frac{4}{100}\left(\frac{\sqrt{2}}{2}\right)^2 + (0,05)^2 = \frac{2}{100} + \frac{25}{100\cdot 100}$

$$A^2 = \frac{2}{100} + \frac{1}{400} = \frac{8}{400} + \frac{1}{400} = \frac{9}{400} \Rightarrow \underline{A = 0,15\,m}$$

Δ.3 $\Sigma F = -D\cdot x \Rightarrow \Sigma F = -100x$ (SI) , $-0,15 \le x \le +0,15$

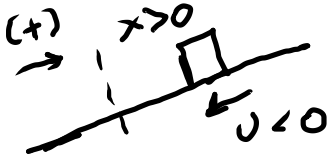


Δ.4

$$\frac{dK}{dt} = \sum F \cdot v = -D \cdot x \cdot v$$

$$K = \frac{E}{2} \Rightarrow \frac{1}{2}(m+M)v^2 = \frac{1}{2}\,\frac{1}{2}(m+M)v_{max}^2 \Rightarrow v = \pm \frac{v_{max}}{\sqrt{2}}$$

2η φορά



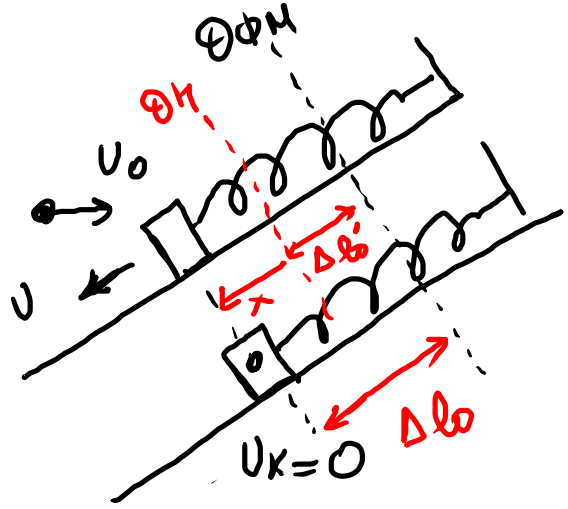
$$v = -\frac{\omega \cdot A}{\sqrt{2}}$$

$$E = K + U \Rightarrow U = \frac{E}{2} \Rightarrow \frac{1}{2}Dx^2 = \frac{1}{2}\,\frac{1}{2}DA^2 \Rightarrow x = +\frac{A}{\sqrt{2}}$$

άρα $$\frac{dK}{dt} = -k\,\frac{A}{\sqrt{2}} \cdot \left(-\frac{\omega A}{\sqrt{2}}\right) = +\frac{100 \cdot 5 \cdot (0,15)^2}{2}$$

$$\Rightarrow \underline{\underline{\frac{dK}{dt} = +\frac{45}{8}\ J/s}}$$

Δ.5



ΑΔΟ στον $x'Ox$

$$P_{\sigma\upsilon\sigma\tau}^{\pi\rho\iota\nu} = P_{\sigma\upsilon\sigma\tau}^{\mu\varepsilon\tau\alpha}$$

$$mU_{0x} - MU = 0$$

$$1 \cdot \frac{4}{3}\sqrt{6} \cdot \frac{\sqrt{3}}{2} - 3U = 0$$

$$U = \frac{2}{9}\sqrt{18} = \frac{2}{3}\sqrt{2}\ m/s$$

για να παραμένει συνεχώς ακίνητο, πρέπει να είναι στη θέση που $\Sigma F_x = 0$, δηλαδή στη θέση του σχήματος.

στην $x = \Delta l_0 - \Delta l_0' = \underline{0,05\ m}$